

Phase Diagram of Two-dimensional Polarized Fermi Gas With Spin-Orbit Coupling

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We investigate the ground state of the two-dimensional polarized Fermi gas with spin-orbit coupling and construct the phase diagram at zero temperature. We find there exist phase separation when the binding energy is low. As the binding energy increasing, the topological nontrivial superfluid phase coexist with topologically trivial superfluid phase which is topological phase separation. The spin-orbit coupling interaction enhance the triplet pairing and destabilize the phase separation against superfluid phase.

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1. INTRODUCTION

The topological properties have been investigated extensively in condensed matter systems such as topological insulators (TIs)[1, 2], topological superconductors (TSCs)[3–6], etc, which are described by topological order [7] instead of the traditional Landau symmetry breaking theory. In ultracold atomic system, the effective spin-orbit coupling (SOC) has been realized recently by utilizing the spatial varying laser fields[8, 9]. With the technique of Feshbach resonance[10, 11], the spin-orbit coupled ultracold atomic systems provide a clean platform to investigate the topological properties of the condensed matter system.

The SOC significantly changes the Fermi surface and largely enhances the low energy density of state[12, 13]. Therefore, many interesting phases and intriguing phenomena become possible. The triplet pairing and the transition temperature are largely enhanced[14] while the pair coherence lengths are suppressed by the SOC[15]. In three dimensions, the ground state of the Fermi system is enriched by the SOC[16–23]. In two dimensions, the superfluid phase of the spin-orbit coupled Fermi gas can be topologically nontrivial[24–27]. Furthermore, there is topologically nontrivial phase separation (TPS) which is the coexistence of superfluid phases with different topological order in the trapped SOC Fermi systems with population imbalance[28, 29].

In this paper, we investigate the uniform polarized two-dimensional (2D) Fermi gas with SOC near a wide Feshbach resonance at zero temperature. The phase separation is possible for a polarized Fermi gas without the SOC due to the competition between the polarization and the pairing interaction. To map out a exact phase diagram, we determined the ground state by minimizing the thermodynamic potential of the phase separation[30].

In the presence of SOC, the Fermi surface is topologically changed. The topological phase transition (TPT) takes place when the excitation gap is closing. Therefore, the topologically nontrivial superfluid phase (TSF) shows up in the phase diagram against the topologically trivial superfluid phase (NSF). For the phase separation phase, the topological phase transition much more tend to take place in the smaller pairing gap component state, thus the phase separation becomes topologically nontrivial.

This paper is organized as follows. In Sec.2, introducing the Hamiltonian of 2D uniform polarized Fermi gas, we obtain the zero temperature thermodynamic potential by mean field theory, and then give the gap equation and the number equations for superfluid phase. In Sec.3, we investigate the ground state by minimizing the thermodynamic potential of the phase separation phase and map out the phase diagram in detail. A brief conclusion is given in Sec.4.

2. FORMALISM OF THE SYSTEM

We consider the uniform 2D polarized Fermi gas with SOC, which is described by the Hamiltonian:

$$H = H_0 + H_{SO} + H_{int}, \quad (1)$$

where H_0 is the kinetic term, H_{SO} is the spin-orbit interaction, and H_{int} is the s-wave interaction between the two fermionic species. They take

$$\begin{aligned} H_0 &= \sum_{\mathbf{k}, \sigma} \xi_{\mathbf{k}, \sigma} c_{\mathbf{k}, \sigma}^\dagger c_{\mathbf{k}, \sigma}, \\ H_{SO} &= \sum_{\mathbf{k}} \lambda k \left(e^{-i\varphi_{\mathbf{k}}} c_{\mathbf{k}, \uparrow}^\dagger c_{\mathbf{k}, \downarrow} + h.c. \right), \\ H_{int} &= -g \sum_{\mathbf{k}, \mathbf{k}'} c_{\mathbf{k}, \uparrow}^\dagger c_{-\mathbf{k}, \downarrow}^\dagger c_{-\mathbf{k}', \downarrow} c_{\mathbf{k}', \uparrow}, \end{aligned} \quad (2)$$

where $\xi_{\mathbf{k}, \sigma} = \hbar k^2 / (2m) - \mu_\sigma$, $c_{\mathbf{k}, \sigma}^\dagger (c_{\mathbf{k}, \sigma})$ denotes the creation (annihilation) operators for a fermion with momentum \mathbf{k} and spin $\sigma = \{\uparrow, \downarrow\}$, λ is the strength of Rashba

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spin-orbit coupling, $\varphi_{\mathbf{k}} = \arg(k_x + ik_y)$, g is the bare s-wave interaction strength which can be renormalized by

$$\frac{1}{g} = - \sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}} + E_b}. \quad (3)$$

By the transformation,

$$\begin{pmatrix} c_{\mathbf{k},\uparrow} \\ c_{\mathbf{k},\downarrow} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\varphi_{\mathbf{k}}} \\ e^{-i\varphi_{\mathbf{k}}} & -1 \end{pmatrix} \begin{pmatrix} a_{\mathbf{k},+} \\ a_{\mathbf{k},-} \end{pmatrix}, \quad (4)$$

the Eq.2 becomes

$$H_0 + H_{SO} = \sum_{\mathbf{k},s=\pm} \left(\xi_{\mathbf{k},s} a_{\mathbf{k},s}^\dagger a_{\mathbf{k},s} - h e^{is\varphi_{\mathbf{k}}} a_{\mathbf{k},s}^\dagger a_{\mathbf{k},-s} \right),$$

$$H_{int} = \sum_{\mathbf{k},s=\pm} \left(\frac{\Delta}{2} e^{is\varphi_{\mathbf{k}}} a_{\mathbf{k},s}^\dagger a_{-\mathbf{k},s}^\dagger + h.c. \right) + \frac{|\Delta|^2}{g}, \quad (5)$$

where $a_{\mathbf{k},\pm}^\dagger(a_{\mathbf{k},\pm})$ is the creation(annihilation) operator for the state with helicity (\pm), $\xi_{\mathbf{k},\pm} = \epsilon_{\mathbf{k}} \pm \lambda k$ with $\epsilon_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$ and the chemical potentials $\mu = (\mu_\uparrow + \mu_\downarrow)/2$, $h = (\mu_\uparrow - \mu_\downarrow)/2$, Δ is the pairing potential which takes $\Delta = g \sum_{\mathbf{k}} \langle c_{-\mathbf{k},\downarrow} c_{\mathbf{k},\uparrow} \rangle$.

The Hamiltonian¹ can be rewritten in the helicity basis $\Psi_{\mathbf{k}} = (a_{\mathbf{k},+}, a_{\mathbf{k},-}, a_{-\mathbf{k},+}^\dagger, a_{-\mathbf{k},-}^\dagger)^T$ as:

$$H = \frac{1}{2} \sum_{\mathbf{k}} \Psi_{\mathbf{k}}^\dagger \mathcal{H}(\mathbf{k}) \Psi_{\mathbf{k}} + \sum_{\mathbf{k}} \xi_{\mathbf{k}} + \frac{|\Delta|^2}{g}, \quad (6)$$

with

$$\mathcal{H}(\mathbf{k}) = \begin{pmatrix} \xi_{\mathbf{k},+} & e^{i\varphi_{\mathbf{k}}} h & \Delta e^{i\varphi_{\mathbf{k}}} & 0 \\ e^{-i\varphi_{\mathbf{k}}} h & \xi_{\mathbf{k},-} & 0 & \Delta e^{-i\varphi_{\mathbf{k}}} \\ \Delta e^{-i\varphi_{\mathbf{k}}} & 0 & -\xi_{\mathbf{k},+} & e^{-i\varphi_{\mathbf{k}}} h \\ 0 & \Delta e^{i\varphi_{\mathbf{k}}} & e^{i\varphi_{\mathbf{k}}} h & -\xi_{\mathbf{k},-} \end{pmatrix}. \quad (7)$$

We know that the classification of above 2D BdG Hamiltonian, which breaks the time-reversal symmetry but preserves the particle-hole symmetry, is Z class [5]. The topological numbers which characterize the topological properties of the superfluid phases are integer. There is topological phase transition at the gap closing point $h = \sqrt{\mu^2 + \Delta^2}$. The topologically nontrivial superfluid phase show up when $h > \sqrt{\mu^2 + \Delta^2}$.

The Hamiltonian can be diagonalized as

$$H = \sum_{\mathbf{k},s=\pm} E_{\mathbf{k},s} \alpha_{\mathbf{k},s}^\dagger \alpha_{\mathbf{k},s} + \frac{1}{2} \sum_{\mathbf{k},s=\pm} (\xi_{\mathbf{k}} - E_{\mathbf{k},s}) + \frac{|\Delta|^2}{g}, \quad (8)$$

where, $\alpha_{\mathbf{k},\pm}^\dagger(\alpha_{\mathbf{k},\pm})$ is the creation(annihilation) operator for the quasiparticles with the excitation spectra $E_{\mathbf{k},\pm} = \sqrt{\xi_{\mathbf{k}}^2 + h^2 + |\Delta|^2 + \lambda^2 k^2} \pm 2E_0$, here $E_0 = \sqrt{h^2(\xi_{\mathbf{k}}^2 + |\Delta|^2) + \lambda^2 k^2}$.

The thermodynamical potential is $\Omega = -\text{Tr} \ln[e^{-\beta H}]$ with $\beta = 1/(k_B T)$. At $T = 0$, the thermodynamical potential is

$$\Omega = \frac{1}{2} \sum_{\mathbf{k},s=\pm} (\xi_{\mathbf{k}} - E_{\mathbf{k},s}) + \frac{|\Delta|^2}{g}. \quad (9)$$

The pairing gap should be self-consistently determined with chemical potential by minimizing the thermodynamic potential $\partial\Omega/\partial\Delta = 0$ and the particle number equations $n_\sigma = -\partial\Omega/\partial\mu_\sigma$. They are given as

$$\sum_{\mathbf{k}} \frac{1}{2\epsilon_{\mathbf{k}} + E_b} = \frac{1}{4} \sum_{\mathbf{k},s=\pm} \frac{1}{E_{\mathbf{k},s}} \left(1 + s \frac{h^2}{E_0} \right), \quad (10)$$

$$n = \frac{1}{2} \sum_{\mathbf{k},s=\pm} \left[1 - \left(1 + s \frac{h^2 + \lambda^2 k^2}{E_0} \right) \frac{\xi_{\mathbf{k},s}}{E_{\mathbf{k},s}} \right],$$

$$pn = -\frac{1}{2} \sum_{\mathbf{k},s=\pm} \frac{h}{E_{\mathbf{k},s}} \left(1 + s \frac{\xi_{\mathbf{k},s}^2 + \Delta^2}{E_0} \right), \quad (11)$$

where, $n = n_\uparrow + n_\downarrow$ is the total particle number and $p = (n_\uparrow - n_\downarrow)/n$ is the polarization. In the presence of SOC, the Fermi surface is topologically changed and the triplet pairing is possible. The condensate fraction should include singlet and triplet contributions $n_c = n_0 + n_1$ which are given as

$$n_0 = 2 \sum_{\mathbf{k}} |\langle c_{\mathbf{k},\uparrow} c_{-\mathbf{k},\downarrow} \rangle|^2$$

$$= \frac{\Delta^2}{8} \sum_{\mathbf{k}} \left[\sum_{s=\pm} \left(1 + s \frac{h^2}{E_0} \right) \frac{1}{E_{\mathbf{k},s}} \right]^2, \quad (12)$$

$$n_1 = \sum_{\mathbf{k}} (|\langle c_{\mathbf{k},\uparrow} c_{-\mathbf{k},\uparrow} \rangle|^2 + |\langle c_{\mathbf{k},\downarrow} c_{-\mathbf{k},\downarrow} \rangle|^2)$$

$$= \frac{\Delta^2}{16} \sum_{\mathbf{k}} \left[\left(\sum_{s=\pm} \frac{s}{E_{\mathbf{k},s}} \right)^2 \sum_{s=\pm} \frac{\lambda^2 k^2 (\xi_{\mathbf{k}} + sh)^2}{E_0^2} \right] \quad (13)$$

3. THE PHASE DIAGRAM IN $p - \lambda k_F/E_F$ PLANE

There is no guarantee that the ground state of the polarized Fermi gas corresponds to one of the spatially homogeneous states. As the competition between the population imbalance and the pairing interaction, the phase separation becomes possible. For the polarized Fermi gas, the stability of the phase separation against the superfluid should be considered like the case without SOC. By introducing the mixing coefficient x ($0 \leq x \leq 1$) and ignoring the interfaces energy between the two coexisting phase, the thermodynamic potential of the phase separation can be written as

$$\Omega = x\Omega(\Delta_1) + (1-x)\Omega(\Delta_2), \quad (14)$$

where, Δ_i ($i = 1, 2$) is the pairing gap of the i component separated state. The thermodynamic potential should be minimized with Δ_i and the mixing coefficient x . The number equations become $n_\sigma = xn_\sigma(\Delta_1) + (1-x)n_\sigma(\Delta_2)$. By solving the gap equations and the number equations

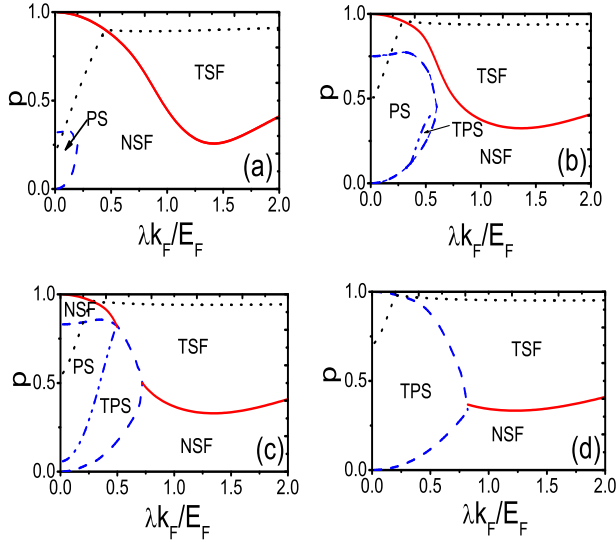


FIG. 1: The phase diagrams in $p-\lambda k_F/E_F$ plane with binding energy (a) $E_b = 0.1E_F$; (b) $E_b = 0.5E_F$; (c) $E_b = 0.6E_F$; (d) $E_b = 1.0E_F$. Here, $E_F = k_F^2/2m = n\pi/m$. The red solid lines separate the TSFs from NSFs phases. The blue dash lines separate the phase separation from superfluid phase. The blue dash-dot-dot lines are the boundaries between topologically trivial and nontrivial phase separation. The dot lines denote the $\Delta/E_F = 10^{-3}$, above which the pairing gap is lower than 10^{-3} .

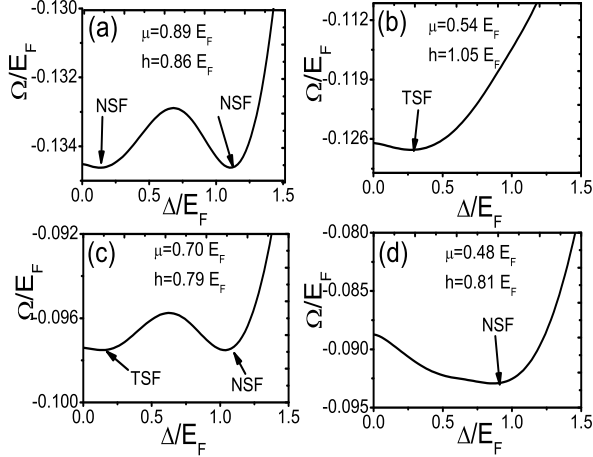


FIG. 2: The thermodynamic potential Ω as a function of the pairing gap Δ with the binding energy $E_b/E_F = 0.6$ for (a) $\lambda k_F/E_F = 0.4$, $p = 0.8$; (b) $\lambda k_F/E_F = 0.7$, $p = 0.8$; (c) $\lambda k_F/E_F = 0.4$, $p = 0.3$; (d) $\lambda k_F/E_F = 0.7$, $p = 0.3$.

selfconsistently, we construct the phase diagram in $p-\lambda k_F/E_F$ plane for different binding energy.

First, we give the phase diagrams in $p-\lambda k_F/E_F$ plane with different binding energy in Fig.1((a) $E_b = 0.1E_F$; (b) $E_b = 0.5E_F$; (c) $E_b = 0.6E_F$; (d) $E_b = 1.0E_F$). The phase separation, which is coexistence of distinct topologically trivial superfluid phases, show up in the absence

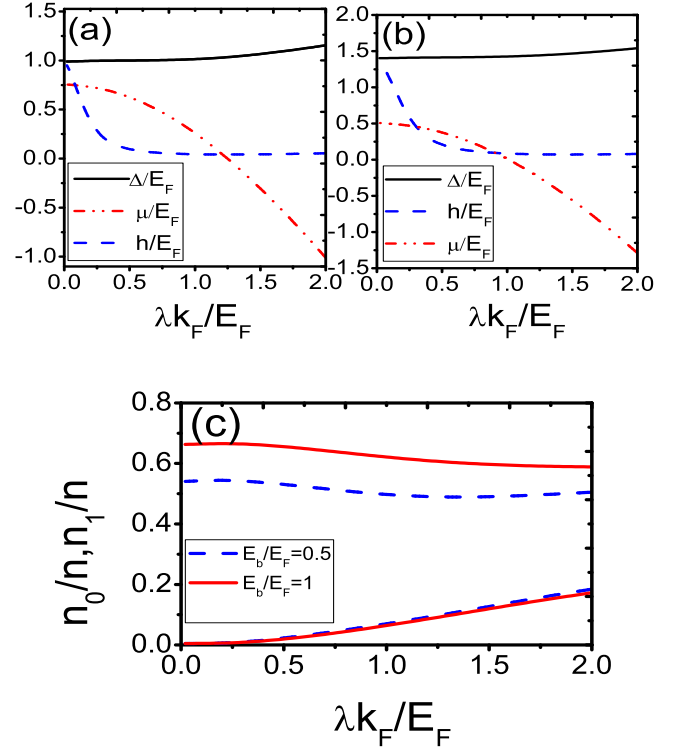


FIG. 3: The pairing gap and the chemical potential as functions of the SOC strength $\lambda k_F/E_F$ with $p = 0.01$ for (a) $E_b = 0.5E_F$ and (b) $E_b = 1.0E_F$. (c) The condensate fractions of singlet and triplet contribution as functions of the SOC for $E_b/E_F = 0.5, 1$ with $p = 0.01$. The above two lines are singlet contributions while the others are triplet contribution. The triplet contributions are enhanced by the SOC.

of SOC. When the polarization is larger than 0.32, the phase separation can not sustain against topologically trivial superfluid (NSF) in the phase diagram without SOC for $E_b = 0.1E_F$ case. The critical polarization increase with the binding energy as shown in Fig.1. This consist with the recent result without the SOC [31].

In the presence of SOC, the Fermi surface is topologically changed and other interesting topologically nontrivial phases are possible. There is topological phase transition when the excitation gap closing at the critical point $h = \sqrt{\mu^2 + \Delta^2}$. The topological phase transition tend to take place in the high polarization area in which the pairing gap is low and the imbalance of the chemical potential is large. Therefor, the phases are TSF in the phase diagrams with high polarization as shown in Fig.1.

For the phase separation phase, the topological phase transition much more tend to take place in the low pairing gap component state. The phase separation become topologically nontrivial when the low pairing gap component state become topologically nontrivial as shown in Fig.1(b),(c),(d). As the binding energy increasing, the topological phase separation is more pos-

sible. The entire phase separation is topologically trivial with $E_b = 0.1E_F$ (shown in Fig. 1(a)) and nontrivial with $E_b = 1.0E_F$ (shown in Fig. 1(d)). The boundary (the red solid line) between the TSF and NSF merge with the phase separation boundary (the blue dash line) as the binding energy increasing. Fig. 1 also show that the SOC destabilize the phase separation against superfluid phase. When the SOC strength increase to a critical value, the phase separation disappear.

Second, we show the behavior of thermodynamic potential toward the pairing gap in different phase regions of the phase diagram for $E_b/E_F = 0.6$ in Fig. 2. The thermodynamic potential has two degenerate minimums in the phase separation regions as shown in Fig. 2(a)(c). The two distinct superfluid phases can show up and coexist in the phase diagram. The two coexistent states are all topologically trivial in Fig. 2(a). But, the smaller component state is topologically nontrivial while the other is topologically trivial in Fig. 2(c). The thermodynamic potential in the superfluid region only has one minimum as shown in Fig. 2(b)(d).

Finally, we show the variation of Δ , μ , h and the condensate fractions for very low polarization ($p = 0.01$) with $E_b = 0.5E_F$ and $E_b = 1E_F$ in Fig. 3. The triplet condensate fractions are enhanced by the SOC. The SOC enhance the triplet pairing in virtue of the topologically change the Fermi surface. Therefore, the system can not sustain the phase separation against the superfluid phase as the triplet pairing increasing as well as the SOC strength.

It should be point out that the gap equation divergent as the pairing gap Δ reduce to zero when $\mu < -(\lambda^4 + 4h^2)/(4\lambda^2)$ or $\mu < \min(-|h|/2, -\lambda^2/2)$, hence there is no boundary between the normal phase and the superfluid phase. We map out the boundaries (dot lines) for $\Delta = 0.001E_F$ as shown in Fig. 1. Above the curve, the pairing gap is $\Delta < 0.001E_F$ and exponentially decreases as the SOC reduce to zero[29].

4. CONCLUSIONS

We construct the phase diagram for the two-dimensional Fermi gas with spin-orbit coupling and population imbalance near a wide Feshbach resonance. We map out the stability regions of the topologically trivial and nontrivial superfluid phase, and phase separation in detail. As the spin-orbit coupling increasing, there is topological phase transition. Therefore, the topologically nontrivial phase separation is possible. The spin-orbit coupling enhance the triplet pairing and suppress the phase separation. The phase separation can not sustain against superfluid phase when the spin-orbit coupling is large.

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